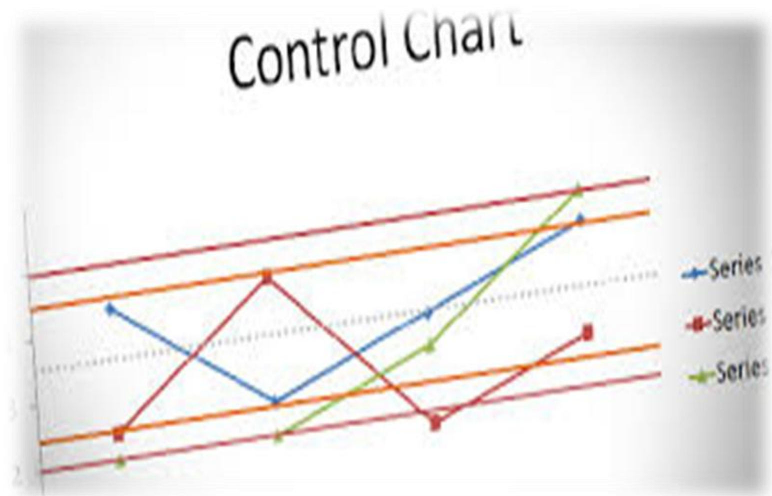
Statistical Process Control

BPF2123 – Quality Management System



Chapter Outline

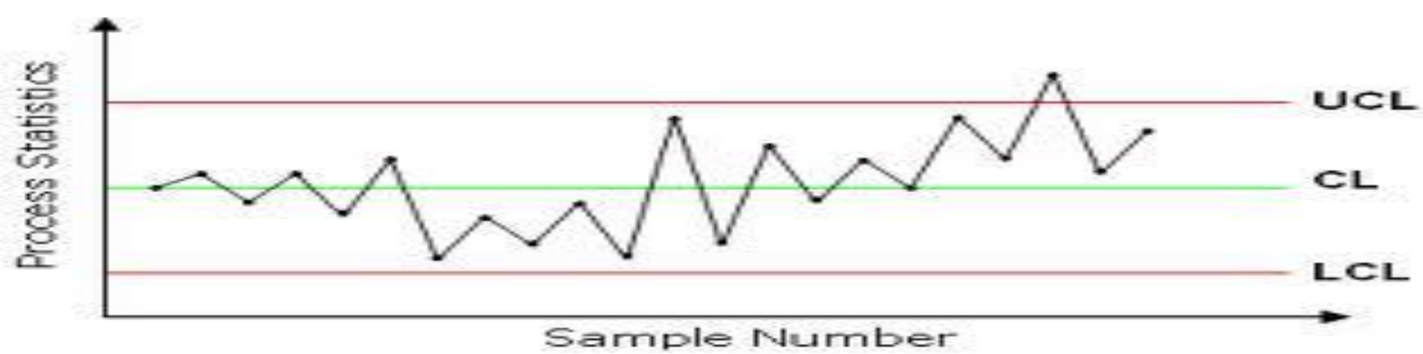
- What is Control Chart?
- Basic Principles
- Analysis of Patterns
- Control Chart Functions
- Variation
- Types of Data
- Control Limits
- Variables Control Charts
- Process Capability

Lesson Outcomes

- Understand the variation concept, control chart function and process capability
- Explain the statistical basis of the control chart
- Solve problems or issues arise in the industries by applying statistical process control technique

What is Control Charts?

- A control chart is a graph with limit lines, called control lines. There are basically three kinds of control lines.
- The chart contains:
  - Center line that represents the average value of the quality characteristics corresponding to the in-control state
  - Two other horizontal lines, called the upper control limit (UCL) and the lower control limit (LCL)
- All the sample points on the control chart are connected with straight-line segments, so that it's easier to visualize how the sequence of points has evolved over time



Basic Principles

- If the process is in control, nearly all of the sample points will fall between chosen control limits and no action is necessary
- However, a point that plots outside of the control limits is interpreted as evidence that the process is out of control, investigation and corrective action are required to find and eliminate the causes
- Even if all the points plot inside the control limits, if they behave in a systematic or non random manner, then this could be an indication that the process is out of control
- If the process is in control, all the plotted points should have an essentially random pattern

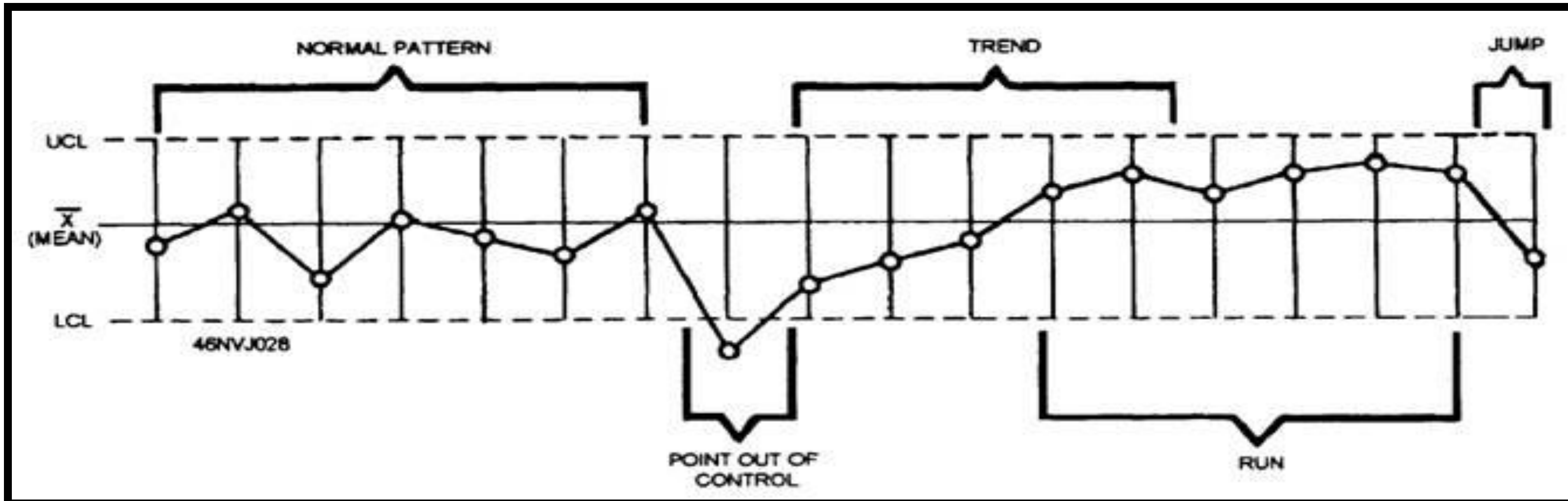
Analysis of Patterns

A control chart exhibits a state of control when:

1. 2/3 of the points are near the center value
2. A few of the points are on or near the center value
3. The points appear to float back and forth across the centerline
4. The points are balanced (in roughly equal numbers) on both sides of the centerline
5. There are no points beyond the control limits
6. There are no patterns or trends on the chart

A process that is not under control or is unstable, displays patterns of variation - the process need to be investigate and determine if an assignable cause can be found for the variation

Analysis of Patterns



Patterns in Control Chart:

- Oscillating
- Change / Jump / Shift
- Runs
- Recurring Cycles
- Freaks / Drift

Control Chart Functions

- A control chart enhances the analysis of the process by showing how the process is performing over time
- Serve 2 basic functions:

1. Control charts are decision making tools

   Provide an economic basis for making a decision as to whether to investigate for potential problems; to adjust the process or to leave the process alone

2. Control charts are problem-solving tools

   Point out where improvement is needed and help to provide a statistical basis on which to formulate improvement actions

Control Chart Functions

i. Monitor and control the machines (parameter; speed, rpm, pressure) and processes
ii. Obtain information about specification and manufacturing
iii. Obtain data about a production run
iv. Supply information (i.e; mean of process, process capability) to customers of conformance to specification
v. Provide a visual baseline for continuing process improvement over time

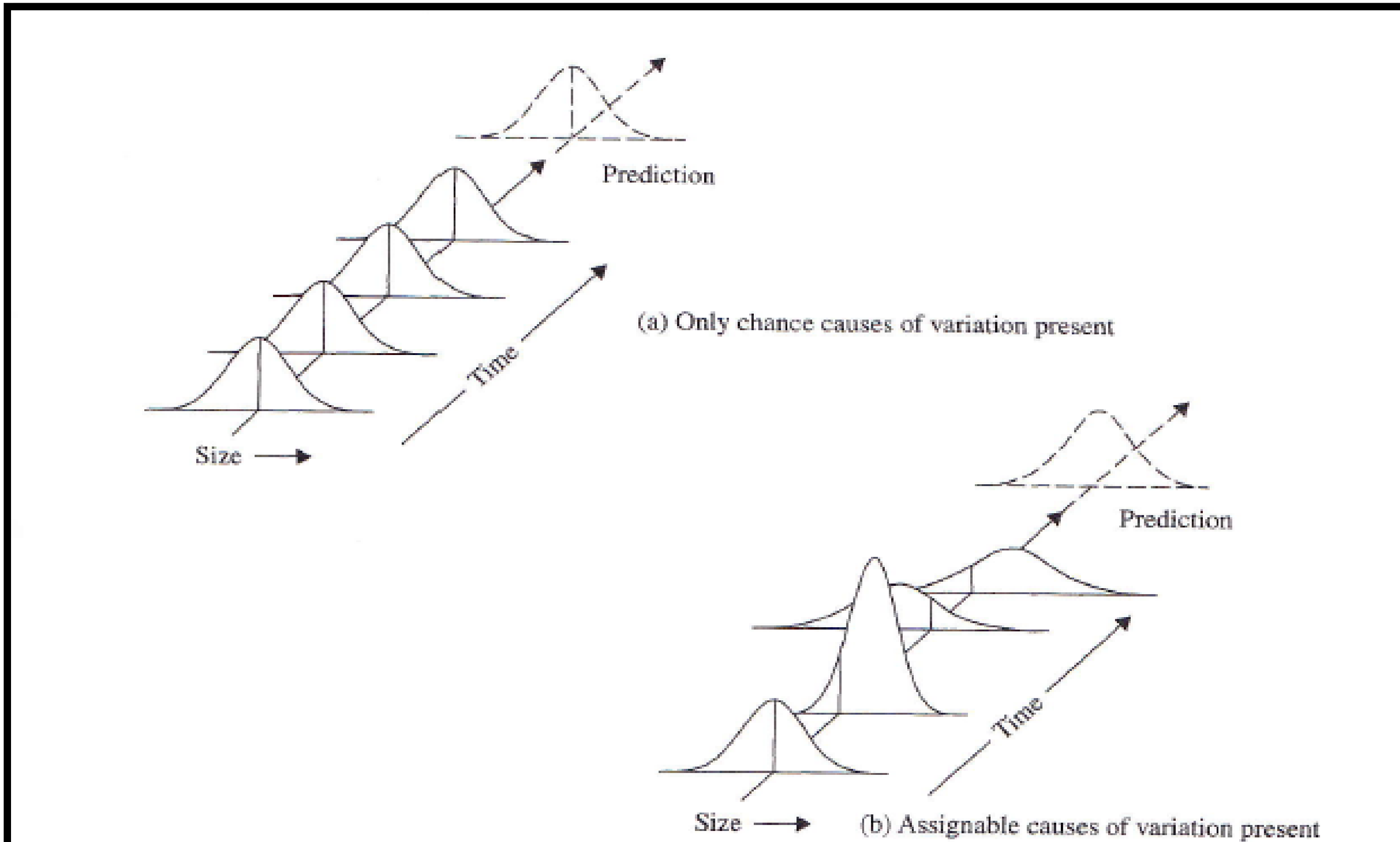
Variation

- Definition : Where no two items / services are exactly the same
- The goal of most processes is to produce products or provide services that exhibit little or no variation
- Several types of variation are tracked with statistical methods:

1. Within-piece : variation within a single item or surface
2. Piece-to-piece : variation that occurs among pieces produced at approximately the same item
3. Time-to-time : variation in the product produced at different times of the day

- Variation in a process is studied by sampling the process; understanding variation and its causes results in better decisions

Variation



Stable and Unstable Variation

Variation

- Chance / Common Causes
  - small random changes in the process that cannot be avoided – due to the inherent variation present in all processes
  - only removable by making a change in the existing process – involves management intervention
  - Example : temperature in the factory building

- Assignable Causes
  - variations in the process that can be identified as having a specific cause (causes that are not part of the process on a regular basis)
  - variation arises because of specific circumstances
  - Example : changes in the thickness of incoming raw material, broken tool
  - Source of variation : process itself, the material used, the operator's actions or the environment

Types of Data

- Two Types :

  i. Variable data – measurable, data derived from incremental measurements, recorded in numerical data, usually continuous measurement.

     Eg : length, voltage, viscosity, weight, time

  ii. Attribute data – character, evaluated with a discrete choice, often collected in the final inspection when assembled products are performance tested / inspected virtually.

     Eg : good/bad, yes/no, accept/reject

Choice Of Control Limits

- 99.73% of the data under a normal curve falls within $\pm 3\sigma$; because of this, control limits are established at $\pm 3\sigma$ from the centerline of the process
- Let w be a sample statistic that measures some quality characteristic of interest, and suppose that the mean of w is μ_w and the standard deviation of w is σ_w

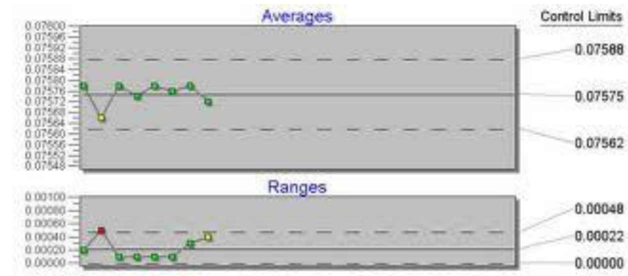
- Formula:

  Upper Control Limit (UCL) $= \mu_w + 3\sigma_w$

  Center Line $= \mu_w$

  Lower Control Limit (LCL) $= \mu_w - 3\sigma_w$

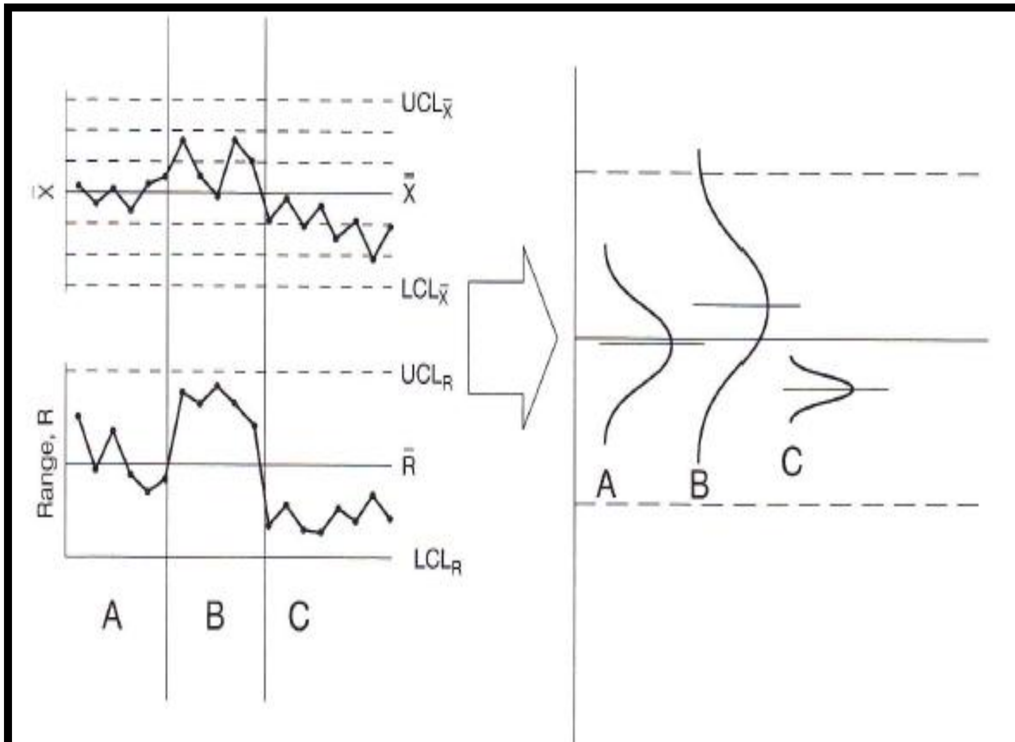
X Bar - R Chart



- The $\dot{x}$ chart is used to monitor the variation of the subgroup averages that are calculated from the individual sampled data
- Averages rather than individual observations are used on control charts because average values will indicate a change in the amount of variation much faster than will individual values
- Control limits on this chart are used to evaluate the variation from one subgroup to another
- The Range chart is a method of determining the amount of variation in the individual samples
- The calculation of the spread of the measurements is necessary to determine whether the parts being produced are similar to one another or not

X Bar - R Chart

The ẋ and range charts are used together in order to show both the center of the process measurements (accuracy) and the spread of the data (precision)



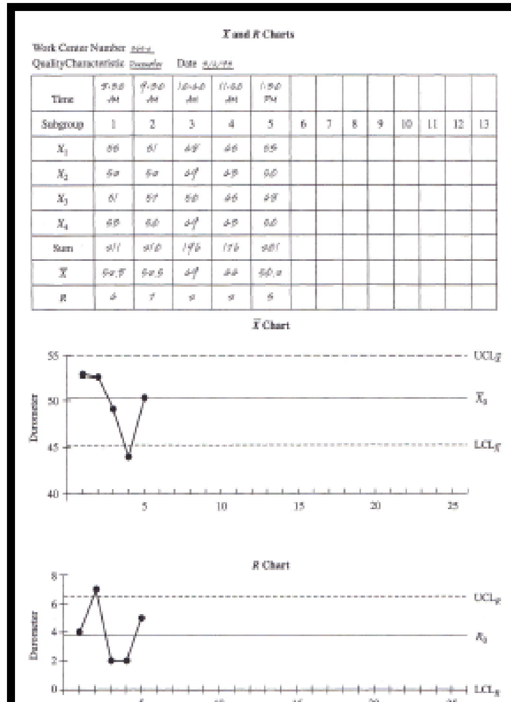


Figure 18-18 Example of a Method of Reporting Inspection Results

X Bar Chart

Determine Centerline

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{m} \bar{X}_i}{m}$$

$\bar{\bar{X}}$ = average of the subgroup averages
$\bar{X}_i$ = average of the ith subgroup
m = number of subgroups

Centerline of the control chart is the process average

Determine Control Limits

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{x}}$$
$$LCL_{\bar{X}} = \bar{\bar{X}} - 3\sigma_{\bar{x}}$$

UCL = upper control limit of the $\bar{X}$ chart
LCL = lower control limit of the $\bar{X}$ chart
$\sigma_{\bar{x}}$ = population standard deviation of the subgroup averages

A2 : the factor that allows the approximation

$$A_2\bar{R} \approx 3\sigma_{\bar{x}}$$

Formulas :

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$
$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

Range Chart

Determine Centerline

$$\bar{R} = \frac{\sum_{i=1}^{m} R_i}{m}$$

$\bar{R}$ = average of the ranges
R_i = individual range values for the sample
m = number of subgroups

Remark :

If product displays a wide spread or a large range, then the individuals being produced are not similar to each other

Determine Control Limits

$$UCL_R = \bar{R} + 3\sigma_R$$
$$LCL_R = \bar{R} - 3\sigma_R$$

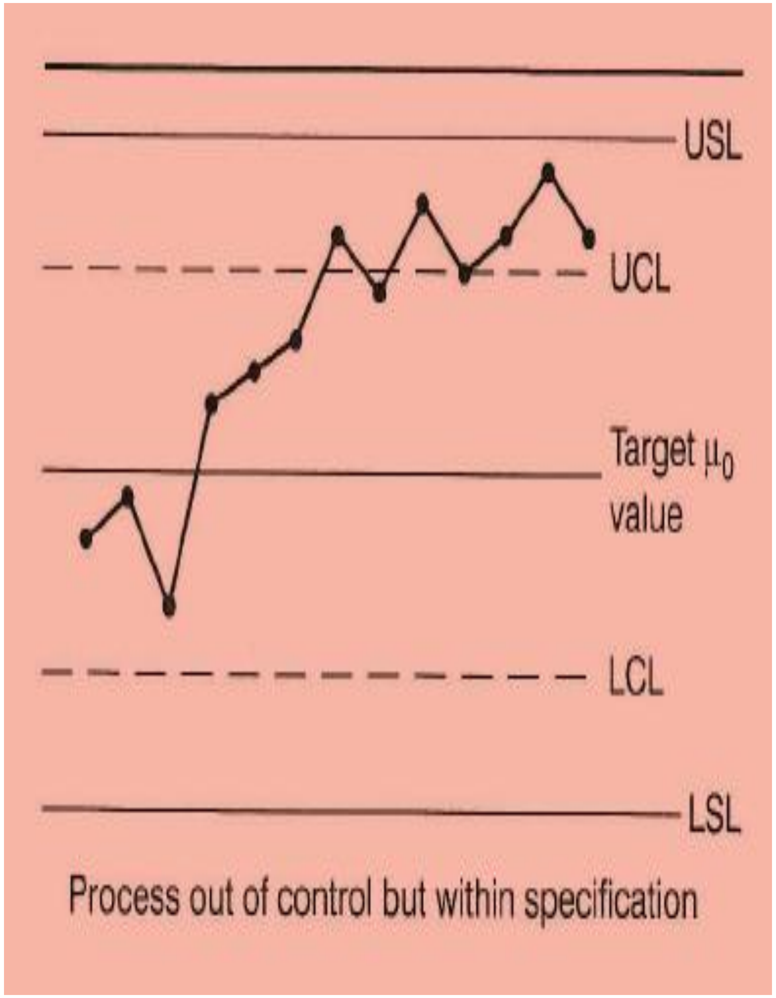
UCL_R = upper control limit of the R chart
LCL_R = lower control limit of the R chart
σ_R = population standard deviation of the subgroup ranges

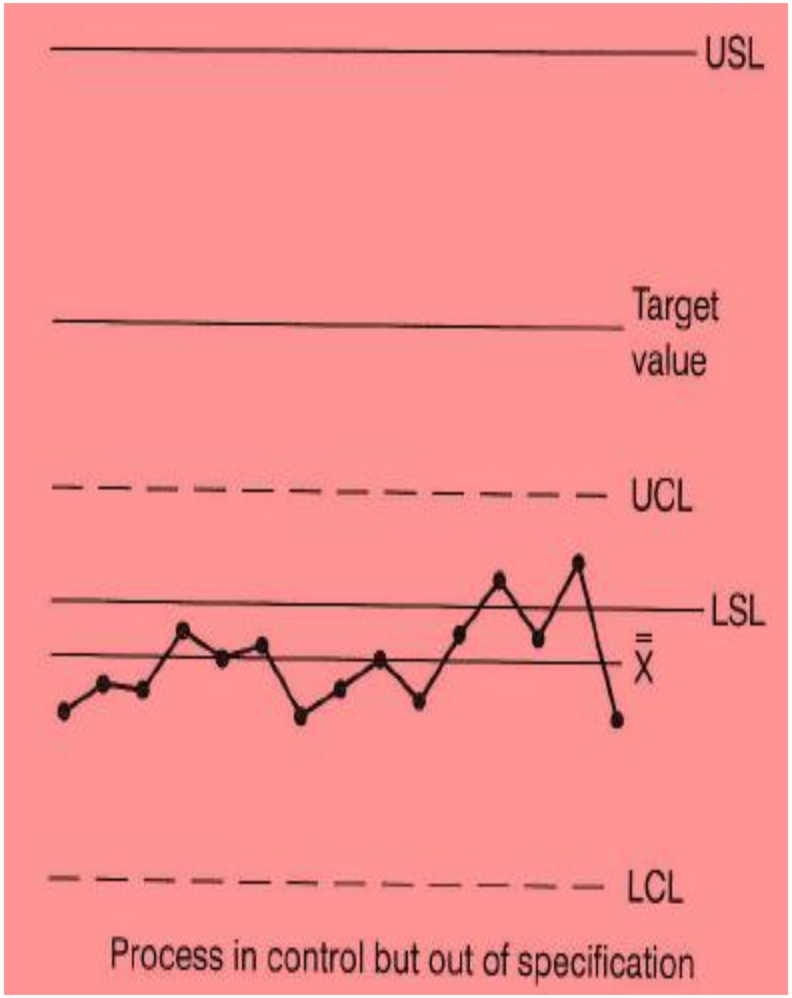
$$UCL_R = D_4\bar{R}$$
$$LCL_R = D_3\bar{R}$$

Control Limit vs. Spec. Limit

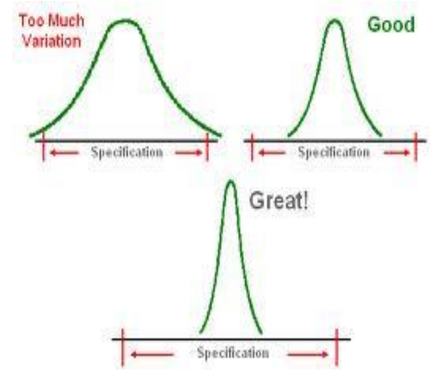
- It is important to note that a process in statistical control will not necessarily meet specifications as established by the customer
- There is a difference between a process conforming to specifications and a process performing within statistical control
- Specifications communicate what the customers expect, want or need from the process – considered the voice of the customer
- Control limits are the voice of the process – a prediction of the variation that the process will exhibit in the near future
- Difference : specifications relay wishes and control limits tell of reality

Control Limit vs. Spec. Limit





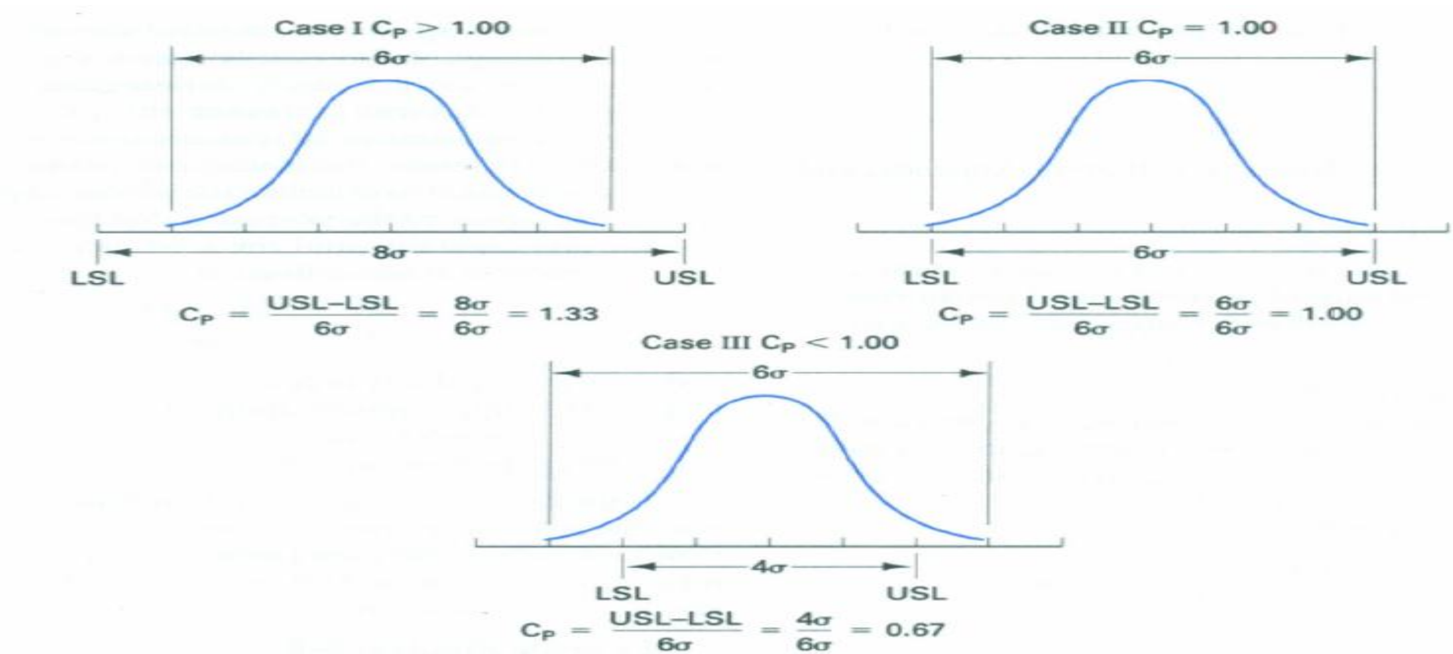
Capability Indices



Capability Index, C_p

$$C_p = \frac{USL - LSL}{6\hat{\sigma}}$$

Cp > 1.00 : Case I situation exists. The greater this value, the better
Cp = 1.00 : Case II situation exists. This is not optimal, but it is feasible
Cp < 1.00 : Case III situation exists. Value of less than 1 are undesirable and reflect the process's inability to meet the specification



Capability Indices

Capability Ratio, C_r

$$C_r = \frac{6\hat{\sigma}}{USL - LSL}$$

Cr < 1.00 : Case I
Cr = 1.00 : Case II
Cr > 1.00 : Case III

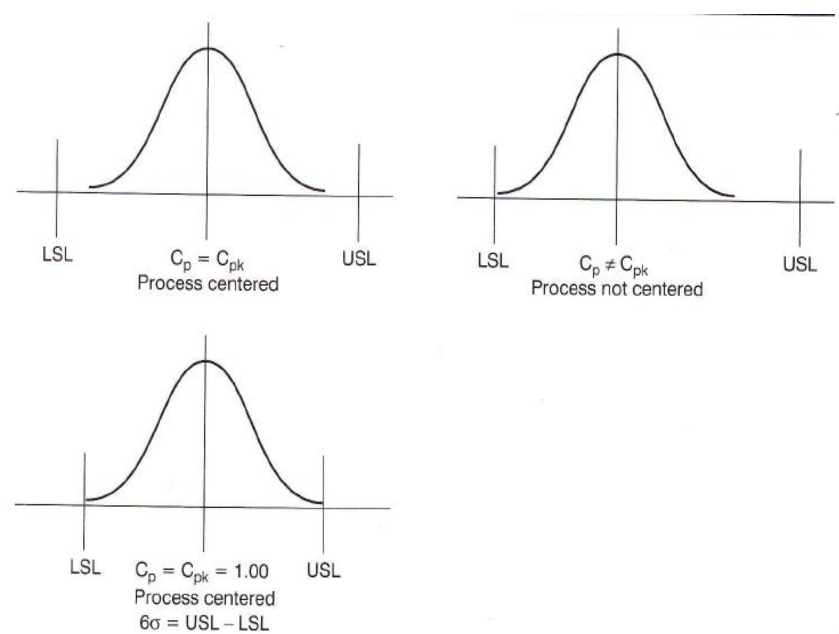
Centering of the Process, C_{pk}

$$C_{pk} = \frac{Z(\min)}{3}$$

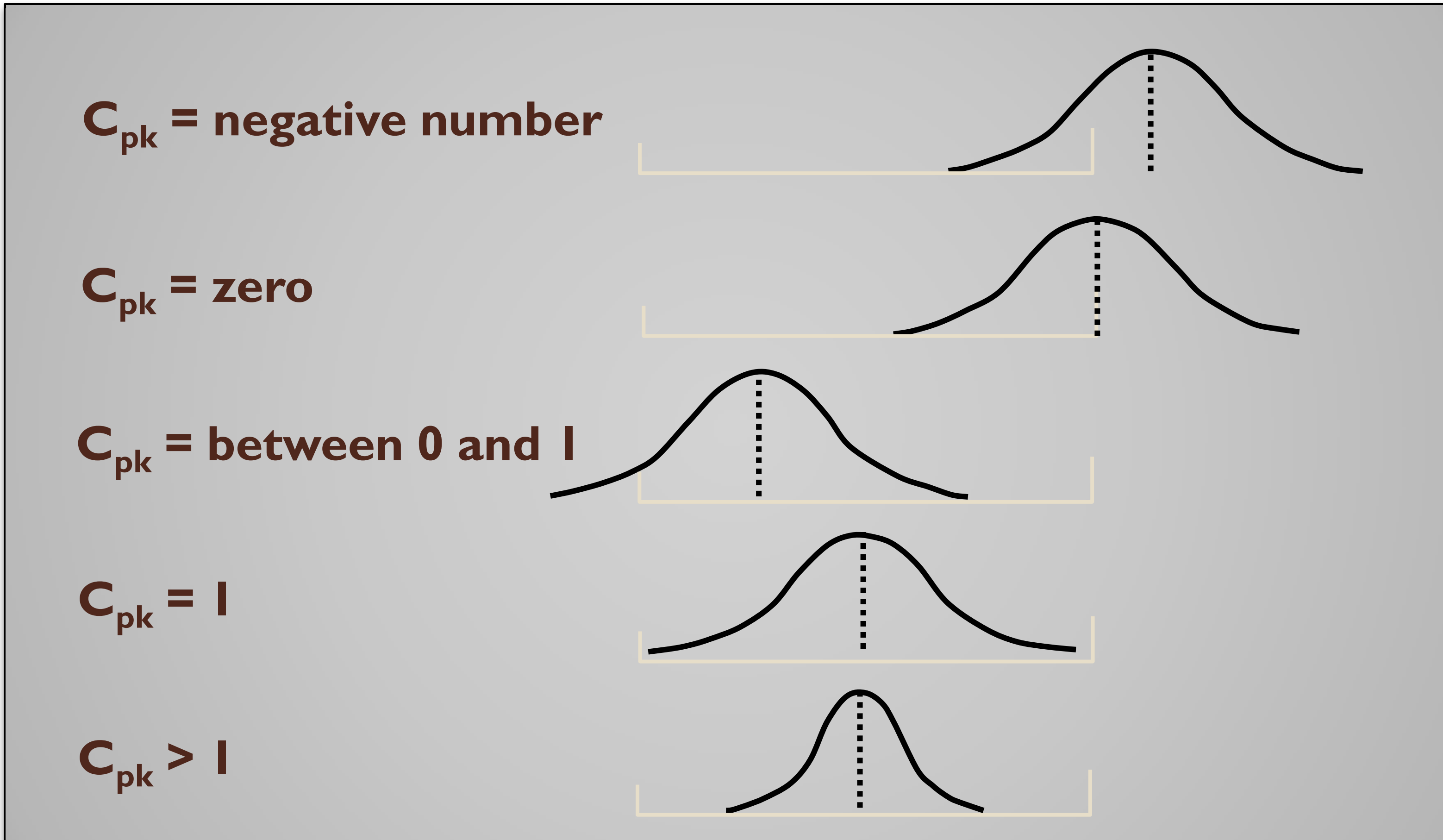
Where Z (min) is the smaller of :

$$Z(USL) = \frac{USL - \overline{X}}{\hat{\sigma}}$$

$$\text{or } Z(LSL) = \frac{\overline{X} - LSL}{\hat{\sigma}}$$



Meanings of C_{pk} Measures

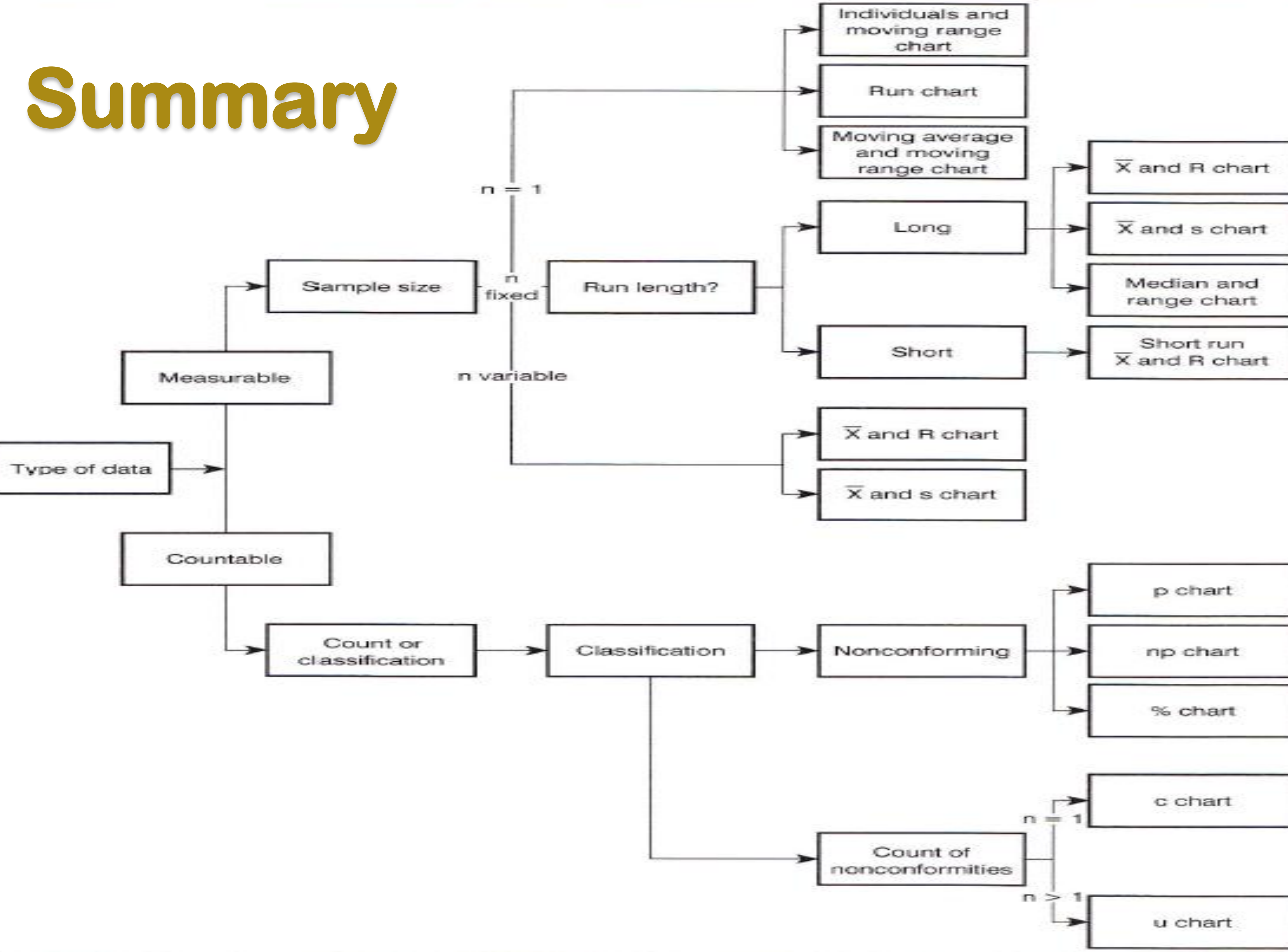


Note : C_{pk} is the ratio that reflects how the process is performing in relation to a nominal, center or target value.

Relationship Between C_p & C_{pk}

1. When C_p has a value of 1.0 or greater, the process is producing product capable of meeting specifications
2. The C_p value does not reflect process centering
3. When the process is centered $C_p = C_{pk}$
4. C_{pk} is always less than or equal to C_p
5. When C_p is greater than or equal to 1.0 and C_{pk} has a value of 1.00 or more, it indicates the process is producing product that conforms to specifications
6. When C_{pk} has a value less than 1.00, it indicates the process is producing product that does not conform to specifications
7. A C_p value of less than 1.00 indicates that the process is not capable
8. A C_{pk} value of zero indicates the process average is equal to one of the specification limits
9. A negative C_{pk} value indicates that the average is outside the specification limits

Summary



Type of data
Measurable
Countable
Sample size
n = 1
n fixed
n variable
Run length?
Long
Short
Individuals and moving range chart
Run chart
Moving average and moving range chart
$\overline{X}$ and R chart
$\overline{X}$ and s chart
Median and range chart
Short run $\overline{X}$ and R chart
$\overline{X}$ and R chart
$\overline{X}$ and s chart
Count or classification
Classification
Nonconforming
p chart
np chart
% chart
Count of nonconformities
n = 1
n > 1
c chart
u chart